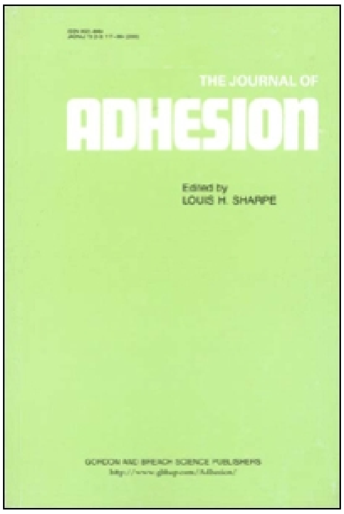

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Non Linear Fracture Mechanics for Adhesive Lap Joints

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The Rice Cherepanov J is calculated for a lap joint in pure shear. By choosing as a condition for fracture a critical value J_c of this quantity the fracture load of the joint is calculated for linear elastic, perfectly plastic and linear hardening behavior of the adhesive. Comparisons are given with experiments with various adhesives and overlap lengths.

KEY WORDS fracture mechanics; elastoplasticity; adhesive joint; overlap length; crack propagation; Rice Cherepanov J integral; theory; experiment.

1 INTRODUCTION

The damage of adhesive joints often results from the initiation of cavities which coalesce to create a crack. A cohesive fracture results from the propagation of such cracks.[1] It can be predicted using fracture mechanics and this is, indeed, what was attempted by Anderson, Bennett and DeVries[2] and Kinloch and Shaw[3] in the framework of linear elasticity and also Yamada[4,5] who studied extension in elastoplasticity.

An experimental device often adopted uses a single or double lap joint loaded in shear. The simplified analysis of Volkersen[6] allows one under those conditions, by neglecting the bending of the adherends which remain elastic, to calculate the shear stress in the adhesive. The Rice Cherepanov integral can then be calculated and this is what we intend to show in order to provide a method of evaluation of the resistance of the joint to crack propagation. Indeed, it is known that when a crack propagates without deviation the Rice Cherepanov integral J is exactly equal to the strain energy release rate G in linear elasticity. It is usually assumed that this prop-

*Corresponding author.

erty remains in elastoplasticity. The propagation condition corresponds then to a critical value of J called J_c.

This model will be compared with experimental results. It will allow one to study the influence of various parameters useful in optimizing the design of adhesive joints.

2 CALCULATION OF RICE CHEREPANOV INTEGRAL J

2.1 General Case

Let us consider a lap joint of length 2l, width B, thickness h, with two adherends of thicknesses and Young's moduli, respectively, h_1 and h_2, E_1 and E_2. A force F is applied on each arm (Figure 1).

The origin of the coordinate, x, is at the center of the joint. Neglecting the bending of the adherends, the adhesive is loaded in pure shear τ. The equilibrium of any section yields

$$h_1\sigma_1 + h_2\sigma_2 = F/B \tag{1}$$

where σ_1 and σ_2 are the normal stresses in the adherends (constant in adherend thickness). The equilibrium of a slice in the adherends yields

$$\tau(x) = h_1\frac{d\sigma_1}{dx} = -h_2\frac{d\sigma_2}{dx} \tag{2}$$

τ being the shear stress in the adhesive.

We consider a crack starting at one end of the joint. A contour ABCDEF along the joint is chosen as shown on Figure 2. The Rice Cherepanov integral J is given by

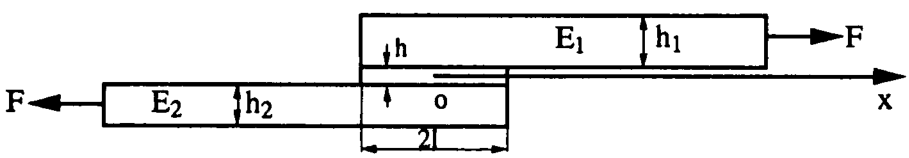

FIGURE 1 Sketch of the lap joint.

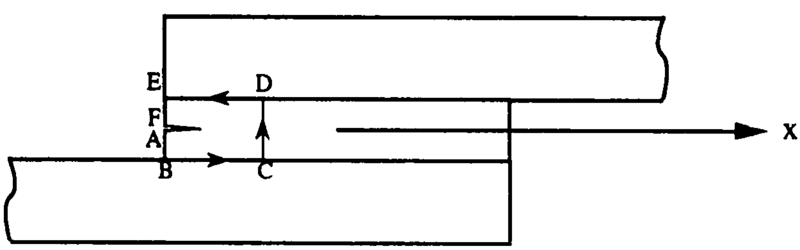

FIGURE 2 Contour for the calculation of the J integral.

$$J = \int_{ABCDEF} W(x)dy - \left(t_x \frac{du_x}{dx} + t_y \frac{du_y}{dy}\right) ds \tag{3}$$

where W(x) is the strain energy density, t_x and t_y are the components of the stress on the contour, and u_x and u_y those of the displacement. It reduces to

$$J = -\int_{BC} t_x \frac{du_x}{dx} dx + \int_{CD} W(x)dy - \int_{DE} t_x \frac{du_x}{dx}(-dx) \tag{4}$$

or

$$J = \int_{BC} \tau(x) \frac{\sigma_2}{E_2} dx + W(x)h + \int_{DE} \tau(x) \frac{\sigma_1}{E_1} dx \tag{5}$$

because

$$\frac{du_1}{dx} = \frac{\sigma_1}{E_1} \text{ and } \frac{du_2}{dx} = \frac{\sigma_2}{E_2} \tag{6}$$

Using the relations (2) and the boundary conditions $\sigma_2(-½) = F/Bh_2$ and $\sigma_1(-½) = 0$ the preceding integral (5) yields:

$$J = \frac{1}{2}\left(\frac{F}{B}\right)^2 \frac{1}{E_2 h_2} - \frac{1}{2}\frac{h_2}{E_2}\sigma_2^2 - \frac{1}{2}\frac{h_1}{E_1}\sigma_1^2 + W(x)h \tag{7}$$

It is easy to demonstrate that this integral is contour independent whatever the mechanical behavior of the adhesive, by showing that $dJ/dx = 0$. This results from the following:

$$W = \int_0^\tau \tau d\gamma \quad \text{and} \quad \frac{dW}{dx} = \tau\frac{d\gamma}{dx} = \frac{\tau}{h}\frac{d(u_1 - u_2)}{dx} = \frac{\tau}{h}\left(\frac{\sigma_1}{E_1} - \frac{\sigma_2}{E_2}\right) \tag{8}$$

2.2 Symmetrical Case

When the two adherends are identical,

$$h_1 = h_2 = h_s \qquad E_1 = E_2 = E_s$$

We will consider the case where there are two symmetric cracks at each end of the joint leaving a length, 2b, of undamaged adhesive. The calculation of the stress should be made by using an effective half length, b, as the stress relaxes in the cracked region.

In such a case, choosing a contour crossing the origin,

$$J = \frac{1}{4}\left(\frac{F}{B}\right)^2 \frac{1}{E_s h_s} + hW(x = 0) \tag{9}$$

To calculate J it is sufficient to know the value of the strain energy density in the center of the joint. An alternative way to calculate J is to choose a contour which goes through the crack tip $(x = -b)$. In such a case the following expression is found:

$$J = hW(x = -b) \tag{10}$$

In the following, this will be carried on in three cases: linear elasticity, perfect plasticity and linear hardening.

2.3 Explicit Calculations of the Fracture Load for Various Types of Mechanical Behavior

2.3.1 Definitions It will be useful to introduce some non-dimensional parameters.

$$S = F/F_0 \quad \text{where} \quad F_0 = 2Bk(E_s h_s h/2\mu)^{1/2} \tag{11}$$

μ and k being, respectively, the elastic modulus and the yield stress in shear of the adhesive.

This parameter will be called S_L when F is equal to F_L, the limit load of the joint, 2Bbk, and S_F when the crack begins to propagate.

$$S_L = b(2\mu/E_s h_s h)^{1/2} \tag{12}$$

We define also

$$S_c = \left(\frac{2\mu J_c}{hk^2} - 1\right)^{1/2} \tag{13}$$

To calculate the fracture load we assume that the crack begins to propagate when the Rice Cherepanov integral J reaches a critical value, J_c. This is valid insofar as J can be considered as the crack driving force. It is the case if the crack propagates without any deviation whatever the fracture mode, which, in the present case, is pure mode II. In the following, in each case, the formula (9) will be used to calculate J, and by setting $J = J_c$, the fracture load S_F will be obtained.

2.3.2 Linear Elastic Case The result obtained by integration of equations (2) and (6) is well known;[7,8] using the condition of symmetry, the following general solution is easily obtained:

$$\tau = A\text{ch}(\alpha S_L) \tag{14}$$

where $\alpha = x/b$, A can be determined by using the equilibrium condition of the joint. In our case, it is found

$$\tau/k = S\text{ch}\alpha S_L/\text{sh}S_L \tag{15}$$

Formula (9) then yields, insetting the critical value J_c of J and using $W(x=0) = \tau^2/2\mu = k^2 S_F^2/2\mu \text{sh}^2 S_L$.

$$S_F = (1 + S_c^2)^{1/2}\text{th}S_L \tag{16}$$

It is easy to check that, indeed, J = G the strain energy release rate given by the compliance formula.

However, for a not completely brittle adhesive plasticity begins to spread as soon as the shear stress in the adhesive reaches the yield stress, k, at the place where it is the largest, that is to say, near the tip of the crack. This happens when

$$S = S_0 = \text{th}S_L \tag{17}$$

Thus, this situation always occurs before the beginning of crack propagation since $S_0 < S_F$, given by the formulae (16) and (17). The case of a plastically deformed adhesive must now be studied.

2.3.3 Perfect Plasticity The yield stress is reached in the adhesive over a length l_p on each side. Let $l_p = \lambda b$. At first, the central part remains elastic and equation (14) keeps holding. Using the condition, $k = \tau$ $(\alpha = (1-\lambda))$, the shear stress there can be written:

$$\tau/k = \mathrm{ch}\alpha S_L / \mathrm{ch} S_L(1-\lambda) \tag{18}$$

and the equilibrium conditions of the joint allow one to write

$$S = \lambda S_L + \mathrm{th} S_L(1-\lambda) \tag{19}$$

According to relation (9)

$$S_F^2 - S_c^2 = \mathrm{th}^2 S_L(1-\lambda_F) \tag{20}$$

λ_F being the plastic zone size when cracking begins $(J = J_c)$. The two preceding equations (19) and (20) give the relation S_F and the parameters S_L and S_c by elimination of λ_F leading to:

$$S_L = S_F + \mathrm{argth}(S_F^2 - S_c^2)^{1/2} - (S_F^2 - S_c^2)^{1/2} \tag{21}$$

The equation (21) shows that for larger values of S_L, S_F can be given by $(1 + S_c^2)^{1/2}$, predicted by the elastic fracture case.

It ceases to be valid at general yield when $\lambda_F = 1$. Then simply

$$S_F = S_L \tag{22}$$

Figure 3 shows the variation of S_F as a function of S_L.

It is also found that if J_c remains constant when the crack propagates, the propaga-

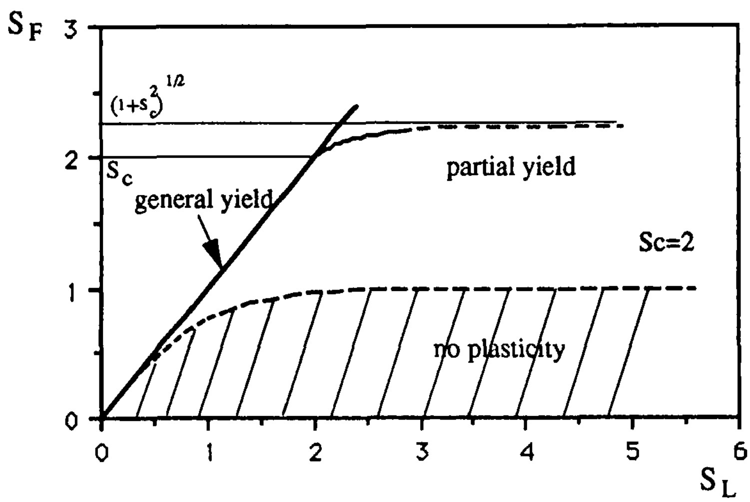

FIGURE 3 Variation of the fracture load S_F as a function of the parameter S_L which is equal to $S_L = b(2\mu/E_s h_s h)^{1/2}$.

tion is catastrophic. S_F is the fracture load of the joint and only small cracks can trigger fracture. Under those conditions the effective length b is nearly equal to l, the half overlap length, saying $b = l - h$, and S_L is indeed a function of this dimension.

2.3.4 Linear Hardening In the case of linear hardening, the mechanical behavior of the adhesive can be written:

$$\tau = \mu_p \gamma + k(1 - \mu_p/\mu) \quad \tau > k \tag{23}$$

where μ_p is the plastic shear modulus and γ the strain.

Let

$$\delta = \mu/\mu_p \tag{24}$$

When the central part of the joint remains elastic, in that area equation (18) keeps holding. However, the equilibrium condition leads to a more complicated equation

$$S = \text{th}[S_L(1-\lambda)] - \delta\text{th}[S_L(1-\lambda)/\delta] + \delta\text{sh}(S_L/\delta)/\text{ch}[S_L(1-\lambda)/\delta] \tag{25}$$

Using the equation (9) again, equation (20) holds:

$$S_F^2 - S_c^2 = \text{th}^2 S_L(1 - \lambda_F)$$

These two preceding equations show that the parameters λ_F can be eliminated when $S = S_F$, giving the relation between S_F and S_L.

When general yield is reached, $\lambda_F = 1$, then

$$S = S_p = \delta\text{sh}(S_L/\delta) \tag{26}$$

and fracture occurs in this case when

$$S = S_F = (S_c^2 + \delta^2)^{1/2}\text{th}(S_L/\delta) \tag{27}$$

Figure 4 shows the variation of S_F as a function of S_L/δ.

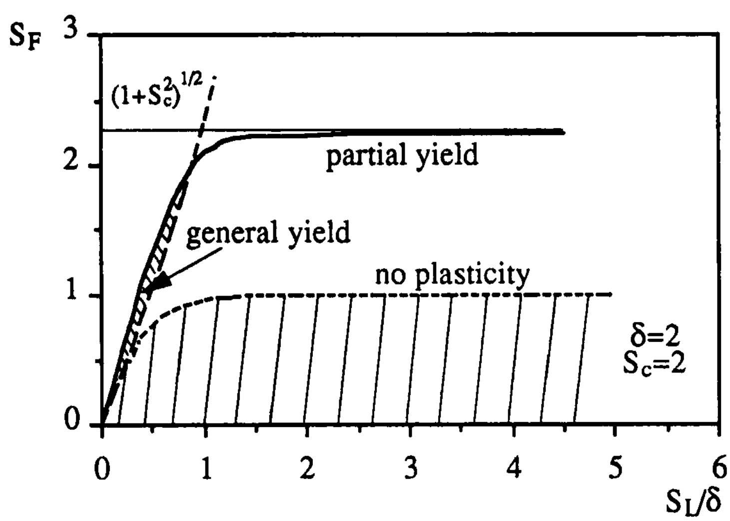

FIGURE 4 Variation of the fracture load S_F and of the general yield load S_p as a function of the parameter S_L/δ equal to $b(2\mu_p/E_s h_s h)^{1/2}$.

3 COMPARISON WITH EXPERIMENTS

Tests were carried out with double lap joints for two of the adhesives (ESP 110, and ECCOBOND 45LV, Emerson and Cuming). The results in the literature given by ESDU[9] were also used for comparisons. The characteristics of the adherends and of the adhesives are given in Table I.

TABLE I
Characteristics of the Joints

joint	h (mm)	h_s (mm)	k (MPa)	E_s (GPa)	μ (MPa)	J_c (kJ/m^2)
ESP 110	0.1	4	36	70	2200	1.2
ECCOBOND	0.1	4	27.2	70	1900	0.65
ESDU	0.1	2	40	140	650	0.51

Table II gives the value of the parameters for the various joints.

TABLE II
Parameters of the Joints

joint	$(2\mu/E_sh_sh)^{1/2}$ (/mm)	Sc	$(1+S_c)^{1/2}$	F_0/B (N/mm)
ESP 110	0.40	6.30	6.38	180
ECCOBOND	0.37	5.69	5.78	147
ESDU	0.21	1.73	2.00	381

The results obtained for various overlap lengths are shown on Figure 5. The solid lines correspond to the model in perfect plasticity. A fair agreement is obtained and it should be noted that other models based on maximum strain (stress) are much less adequate.

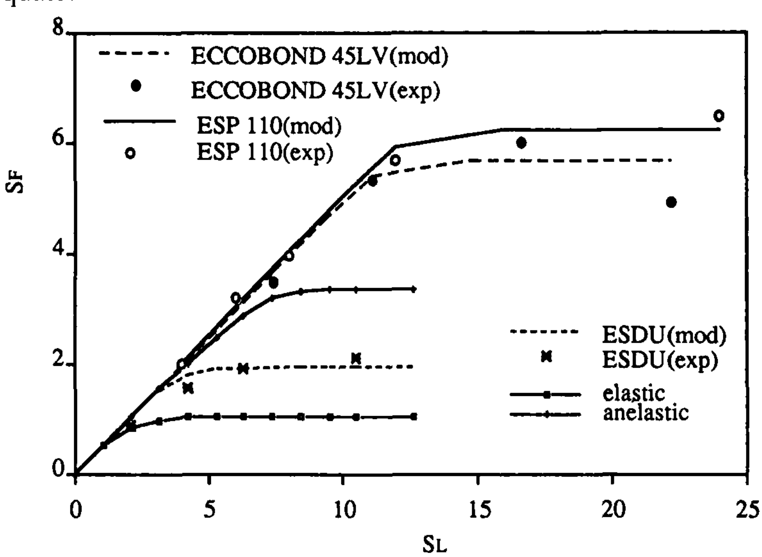

FIGURE 5 Comparison between experimental results obtained for different overlap lengths and the model in perfect plasticity.

4 DISCUSSION

The most important parameters which determine the strength of a lap joint are S_L and S_c. When two joints are such that these non-dimensional parameters are equal, they should behave in the same way. They allow one to study the influence of the mechanical and geometrical parameters.

Figure 5 clearly demonstrates that a certain critical overlap length, l_c, should be reached given by $S_L = S_c$. As it is a large enough number this relation yields

$$l_c(2\mu/E_sh_sh)^{1/2} \approx (2\mu J_c/hk^2)^{1/2}$$
$$l_c = (E_sh_sJ_c)^{1/2}/k \tag{28}$$

When the overlap half length is larger than this critical value, l_c, the fracture load is equal to $2B(E_sh_sJ_c)^{1/2}$ which is independent of the adhesive thickness. The only parameter for a given adherend is then the fracture toughness, J_c, of the adhesive. However, when the critical length is not reached, S_F, which depends on h through the parameter S_L, increases when the adhesive thickness decreases. But again, after general yield, the adhesive thickness does not influence the limit load.

In this analysis the bending of the adherends was neglected. This bending creates normal stresses on the adhesive interface and thus introduces a mode I opening which should add to the crack extension force. This case can be easily studied in the elastic case using the solution given by Goland and Reissner.[10] However, the experimental results and the simple shear theory show that the influence of the bending is slight.

5 CONCLUSION

The Rice Cherepanov integral was calculated for a lap joint loaded in pure shear. As the adherends remain elastic, it is contour independent whatever the mechanical behavior of the adhesive. For linear elastic, perfectly plastic and linear hardening behavior of the adhesive, explicit formulae were given which allow one to determine the fracture load of the joint.

This model was shown to agree well with experiments. This J criterion is different from the maximum strain (or stress) criteria.

The fracture load is essentially a function of the parameters $(2\mu/E_sh_sh)^{1/2}$ and $(2\mu J_c/hk^2)^{1/2}$. Above a critical overlap half length l_c given by $(E_sh_sJ_c)^{1/2}/k$, the fracture load is constant and equal to $2B(E_sh_sJ_c)^{1/2}$, independent of the adhesive thickness.

Table of Symbols

b	uncracked adhesive half length
h	adhesive thickness
k	adhesive shear yield stress
l	overlap half length

l_p	plastic zone size
t_x, t_y	stress components
u_x, u_y	displacement components
x	coordinate
B	joint width
E_1, E_2, E_s	Young's moduli of the adherends
F	applied load
F_0	normalization load $F_0 = 2Bk(E_s h_s h/2\mu)^{1/2}$
F_R	fracture load
G	strain energy release rate in linear elasticity
J	Rice Cherepanov integral
J_c	fracture toughness
S	non-dimensional parameter $S = F/F_0$
S_L	non-dimensional parameter $S_L = b(2\mu/E_s h_s h)^{1/2}$ equal to the value of S at limit load
S_F	value of load parameter at fracture
S_0	value of load parameter at first yielding
S_p	value of load parameter at general yield
S_c	non-dimensional fracture parameter $S_c = (2\mu J_c/hk^2 - 1)^{1/2}$
α	ratio x/l or x/b
δ	modulus ratio $(\mu/\mu_p)^{1/2}$
λ	plastic size ratio $\lambda = l_p/b$
λ_F	value of λ at fracture
σ, σ_1, σ_2	normal stress in adherends
τ	shear stress in adhesive

References

1. G. K. Hu, Ph.D. Thesis, Ecole Centrale Paris (1991).
2. G. P. Anderson, S. J. Bennett and K. L. DeVries, *Analysis and Testing of Adhesive Bonds* (Academic Press, Inc., New York, 1977).
3. A. J. Kinloch and S. J. Shaw, *Developments in Adhesives-2*, A. J. Kinloch, Ed. (Appl. Sci. Publ., London, 1981), p. 83.
4. S. E. Yamada, *Engng. Fracture Mech.* **27,** 315 (1987).
5. S. E. Yamada, *Engng. Fracture Mech.* **29,** 673 (1988).
6. O. Volkersen, *Luftfahrtforschung* **15,** 41 (1938).
7. R. D. Adams and W. C. Wake, *Structural Adhesive Joints in Engineering* (Elsevier Applied Science Publishers Ltd, London. 1984).
8. L. J. Hart-Smith, in *Developments in Adhesives-2*, A. J. Kinloch, Ed. (Appl. Sci. Publ., London, 1981), p. 1.
9. ESDU 79016, Engineering Science Data Unit (1979).
10. M. Goland and E. Reissner, *J. Appl. Mechanics* **11,** 17 (1944).